

Method 3: Homogenous DE.

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right) \quad \text{or} \quad \frac{dy}{dx} = \frac{M(x,y)}{N(x,y)} \quad \begin{matrix} \text{of equal} \\ \text{degree} \end{matrix}$$

each monomial present in M & N also has same degree.

Note: $y = vx$ $\frac{dy}{dx} = v + x \frac{dv}{dx}$

1) $x \frac{dy}{dx} = y (\log y - \log x + 1)$

$$\frac{dy}{dx} = \frac{y}{x} \left(\log \frac{y}{x} + 1 \right)$$

$$v + x \frac{dv}{dx} = v (\log v + 1)$$

$$\cancel{v} + x \frac{dv}{dx} = v \log v + \cancel{v}$$

$$\int \frac{dv}{v \log v} = \int \frac{dx}{x}$$

$$\log v = t$$

$$\frac{1}{v} dv = dt$$

$$\int \frac{1}{t} dt = \log x + \log C$$

$$\log (\log v) = \log x + \log C \Rightarrow \log v = xC$$

$$v = e^{xC}$$

$$\frac{y}{x} = e^{xC}$$

$$y = x e^{xC} \quad \underline{\text{Ans}}$$

2) $(y^2 - x^2) \frac{dy}{dx} = -2xy$

$$\Rightarrow \frac{dy}{dx} = \frac{-2xy}{y^2 - x^2}$$

$$v + x \frac{dv}{dx} = \frac{-2x \cdot vx}{v^2 x^2 - x^2}$$

$$= \frac{-2v}{v^2 - 1}$$

$$x \frac{dv}{dx} = \frac{-2v}{v^2 - 1} - v = \frac{-2v - v^3 + v}{v^2 - 1} = \frac{-(v^3 + v)}{v^2 - 1}$$

$$x \frac{dv}{dx} = \frac{-2v}{v^2-1} - v = \frac{-2v - v^3 + v}{v^2-1} = \frac{-(v^3+v)}{v^2-1}$$

$$= \frac{-v(v^2+1)}{v^2-1}$$

$$\frac{(v^2-1)}{v(v^2+1)} dv > - \frac{dx}{x}$$

$$v^2+1 = t$$

$$2v dv = dt$$

$$\int \left(\frac{2v}{v^2+1} - \frac{1}{v} \right) dv = \int - \frac{dx}{x}$$

$$\log(v^2+1) - \log v = -\log x + \log c$$

$$\log \frac{v^2+1}{v} = \log \frac{c}{x}$$

$$\frac{v^2+1}{v} = \frac{c}{x} \Rightarrow \frac{y^2/x^2+1}{y/x} = \frac{c}{x}$$

$$\frac{y^2+x^2}{\cancel{x^2} \cdot y} \cdot x = \frac{c}{x}$$

$$y^2+x^2 = cy$$

Method IV : Eqⁿ. reducible to homogeneous form.

$$(a_1x + b_1y + c_1) dx + (a_2x + b_2y + c_2) dy = 0$$

$$\frac{dy}{dx} = - \frac{(a_1x + b_1y + c_1)}{a_2x + b_2y + c_2} \quad \begin{cases} x \rightarrow X+h \\ y \rightarrow Y+k \end{cases}$$

$$1) (5x + 2y + 1) dx + (2x + y + 1) dy = 0$$

$$\frac{dy}{dx} = \frac{-(5x + 2y + 1)}{2x + y + 1} = \frac{-(5X + 2Y + \cancel{5h + 2k + 1})}{(2X + Y + \cancel{2h + k + 1})}$$

$$\text{put } 5h + 2k + 1 = 0 \quad \& \quad 2h + k + 1 = 0 \quad (h, k) = (1, -3)$$

$$\frac{dy}{dx} = -\frac{(5x+2y)}{2x+y}$$

$$y = vx \quad \frac{dy}{dx} = v + x \frac{dv}{dx}$$

(2)

$$v + x \frac{dv}{dx} = -\frac{(5+2v)}{(2+v)}$$

$$x \frac{dv}{dx} = -\left(\frac{5+2v}{2+v}\right) - v$$

$$= -\frac{5-2v-2v-v^2}{2+v} = -\frac{(v^2+4v+5)}{2+v}$$

$$\frac{-(5x+2vx)}{2x+vx} = -\frac{x(5+2v)}{x(2+v)}$$

$$= -\frac{(v^2+4v+5)}{2+v}$$

$$\int \frac{2+v}{v^2+4v+5} dv = \int -\frac{dx}{x}$$

$$\int \frac{1}{2t} dt = -\int \frac{dx}{x}$$

$$v^2+4v+5 = t \quad (1)$$

$$(2v+4) dv = dt$$

$$2(v+2) dv = dt$$

$$\frac{1}{2} \log t = -\log x + \log C$$

$$\log t^{1/2} = \log \frac{C}{x}$$

$$\Rightarrow t^{1/2} = \frac{C}{x}$$

$$(1) \Rightarrow v^2+4v+5 = \frac{C^2}{x^2}$$

$$(2) \Rightarrow \frac{v^2}{x^2} + \frac{4v}{x} + 5 = \frac{C^2}{x^2}$$

$$\frac{v^2+4xv+5x^2}{x^2} = \frac{C^2}{x^2}$$

$$x = X+h = X+1$$

$$y = Y+k = Y-3$$

$$X = x-1$$

$$Y = y+3$$

(3)

$$(3) \Rightarrow (y+3)^2 + 4(x-1)(y+3) + 5(x-1)^2 = C^2$$

$$y^2+9+6y + 4xy - 4y + 12x - 12 + 5x^2 + 5 - 10x = C^2$$

$$y^2+4xy+5x^2+2x+2y+2 = C^2$$

$$= C$$

Ans

$$\frac{y + 4xy + 5x^2 + 2x + 1}{y + 2} = C$$

Ans.

≥ 9

h.w. $(2x + 3y + 1) dx + (4x + 6y + 1) dy > 0$